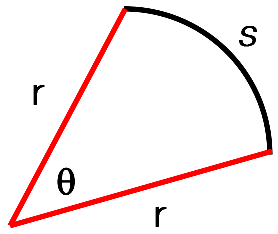
RMIT UNIVERSITY

ENST1.4: ANGULAR MOTION

Arc Length

Generally, any angle at the centre of a circle is defined in radians by the equation

$$\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}, \quad \text{or} \quad s = r\theta$$



In a complete circle, $\theta = 360^\circ$, so

$$\theta = \frac{s}{r} = \frac{2\pi r}{r}, \quad \text{or} \quad 360^\circ = 2\pi \text{ radians}$$

So $\pi = 180°$ and 1 radian = 57.3°

Also, 1 revolution $= 2\pi$ radians . Therefore, the conversions are:

From radians to degrees: $\times \dfrac{180}{\pi}$ From degrees to radians: $\times \dfrac{\pi}{180}$

Angular Velocity ω

Linear Velocity v is the rate of change of displacement, or $v = \dfrac{s}{t} = \dfrac{r\theta}{t}$

Angular Velocity ω is the rate of change of angular displacement, or $\omega = \dfrac{\theta}{t}$

where ω is measure in rad/s.

Therefore, linear and angular velocity are related by the formula

$$v = r\,\omega$$

Converting from r.p.m to radians per second $1\ \text{r.p.m} = \frac{1\,\text{rev}}{1\ \text{min}} = \frac{2\pi}{60\,\text{s}} = \frac{\pi}{30}$

Or:

From r.p.m to radians: $\times \frac{\pi}{30}$ From radians to r.p.m: $\times \frac{30}{\pi}$

Example A weight on the end of a string describes a circular path of radius 0.5m. If its linear velocity is 2m/s what is the angular velocity in (a) rad/s (b) r.p.m?

Solution (a) $\omega = \frac{v}{r} = \frac{2}{0.5} = 4\ \text{rad/s}$ (b) $4\ \text{rad/s} = 4 \times \frac{30}{\pi} = 38.2\ \text{r.p.m}$

Angular Acceleration α

Linear Acceleration a is the rate of change of velocity, or

$$a = \frac{v}{t} = \frac{r\omega}{t} \quad \text{using} \quad v = r\omega$$

Now $\frac{\omega}{t}$ is the rate of change of angular velocity, or *angular acceleration* . Hence

$$a = r\,\alpha$$

It can be shown that equations derived for linear motion apply equally to angular motion provided angular symbols are substituted

Linear Equations	Angular Equations
$s = v\,t$	$\theta = \omega\,t$
$a = \dfrac{v - v_0}{t}$	$\alpha = \dfrac{\omega - \omega_0}{t}$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$v^2 = v_0{}^2 + 2as$	$\omega^2 = \omega_0{}^2 + 2\alpha\theta$
$s = v_0 t + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

Example A flywheel increases in velocity from rest to 400 r.p.m. in 15 seconds. Calculate (a) the angular acceleration, and (b) the number of revolutions it makes.

Solution

(a) $\omega_0 = 0,\ \omega = 400\text{r.p.m.} = \dfrac{400 \times \pi}{30} = 41.9\ \text{rad/s},\ \ t = 15,\ \ a = \ ?$

Use equation $\omega = \omega_0 + \alpha t,$ $\alpha = \dfrac{\omega - \omega_0}{t} = \dfrac{41.9 - 0}{15} = 2.79\,\text{rad/s}^2$

(b) $\theta = ?$ Use equation $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \ = 0(15) + \tfrac{1}{2}(2.79)(15)^2 = 314\,\text{radians}$$

Recall 1 revolution = 2π radians, or 1 radian=1/2π revolutions. Therefore, 314 radians is

$$\frac{1 \times 314}{2\pi} = 50 \text{ revolutions}$$

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Exercise

1. The bob of a pendulum of length 1.5m swings through a 20cm arc. Calculate the angular displacement.

2. A motor revolving at 1500 r.p.m. slows down uniformly to 1200 r.p.m. in 15 seconds. Find (a) the angular acceleration and (b) the linear acceleration of a point 1.5m from the centre.

3. A flywheel operates at 300 r.p.m. Calculate (a) the angular velocity and (b) the linear velocity 1.25 from the centre.

4. Calculate the angular velocity of a car which rounds a curve of radius 10m at 60 km per hr.

Answers

1. 0.13 rad.

2. (a) 6.28 rad s^{-2} (b) 9.4 m s^{-2}

3. (a) 31.42 rad s^{-2} (b) 39.28 m s^{-2}

4. 1.7 rad s^{-1}