RMIT UNIVERSITY

DE2 FIRST ORDER LINEAR

If an equation can be written as

$\frac{dy}{dx} + p(x).y = q(x)$

Then it is termed a linear Differential Equation (DE).

Often the variables will not be separable.

If this is the case, then the integrating factor technique may be utilised to find a solution to the DE:

If $\frac{dy}{dx} + p(x).y = q(x)$

Let $I = e^{\int p(x)dx}$

Then $y\,I = \int \big(I \times q(x)\big)dx$

Solve for y.

Example:

Solve for $y(x)$ given $\frac{dy}{dx} + 5y = e^{2x}$, and $y(0) = 0$

Solution:

Since the equation is in the form: $\frac{dy}{dx} + p(x).y = q(x)$

We let $p(x) = 5$ and $q(x) = e^{2x}$

Since $I = e^{\int p(x)dx}$ and $\int p(x)\,dx = \int 5\,dx = 5x$

Then $I = e^{5x}$

We know the solution in in the form: $y\,I = \int \big(I \times q(x)\big)dx$

Therefore: $ye^{5x} = \int e^{5x} \times e^{2x}\,dx$

$ye^{5x} = \int e^{7x}\,dx$

$ye^{5x} = \frac{1}{7}\,e^{7x} + c$ Dividing through by e^{5x} gives:

$y = \frac{1}{7}\,e^{2x} + ce^{-5x}$

Given $y(0) = 0$, then $y(0) = \frac{1}{7}\,e^{0} + ce^{0} = \frac{1}{7} + c = 0.$ Therefore $c = -\frac{1}{7}$

$y = \frac{1}{7}\,e^{2x} - \frac{1}{7}e^{-5x}$

Exercise

1. $3\frac{dy}{dx} + 12y = 4$
2. $x\frac{dy}{dx} + 2y = 3$
3. $\frac{dy}{dx} + 2xy = x\ ; \quad y(0) = -3$
4. $\frac{dy}{dx} + y = e^{3x}$
5. $y' + 3x^2y = x^2$
6. $x^2y' + xy = 1$
7. $xdy = (x\ \sin x - y)dx$
8. $\cos x\frac{dy}{dx} + y\ \sin x = 1$
9. $x\frac{dy}{dx} + 4y = x^3 - x$
10. $\cos^2 x\frac{dy}{dx} + y = 1\ ; \quad y(0) = -3$

Answers

1. $y = \frac{1}{3} + ce^{-4x}$
2. $y = \frac{3}{2} + cx^{-2}$
3. $y = \frac{1}{2} - \frac{7}{2}e^{-x^2}$
4. $y = \frac{1}{4}e^{3x} + ce^{-x}$
5. $y = \frac{1}{3} + ce^{-x^3}$
6. $y = x^{-1}lnx + cx^{-1}$
7. $y = -\cos x + \frac{\sin x}{x} + cx^{-1}$
8. $y = \sin x + c\cos x$
9. $y = \frac{1}{7}x^3 - \frac{1}{5}x + cx^{-4}$
10. $y = 1 - 4e^{-\tan x}$