PME1.4: ERRORS IN CALCULATIONS

Error in a Calculated Value

When a result is calculated from a number of values, each of which has an uncertainty, then it follows that the calculated result is also subject to uncertainty.

Indeed, errors accumulate – that is, the **uncertainty increases**.

Error in a sum or difference

Suppose a measurement consists of two lengths added together.

$A = 4.0 \pm 0.2 \text{ cm}$ and $B = 2.0 \pm 0.1 \text{cm}$

In adding *A* and *B*, the measured values are added, and also the uncertainties.

$$\therefore A + B = (4.0 + 2.0) \pm (0.2 + 0.1) \text{cm}$$
$$A + B = 6.0 \pm 0.3 \text{cm}$$

If the value that we seek is the difference between two lengths, for example *A* – *B*, then we calculate the difference between the measured values, **but we still add the uncertainties**.

$$\therefore A - B = (4.0 - 2.0) \pm (0.2 + 0.1) \text{cm}$$
$$A - B = 2.0 \pm 0.3 \text{cm}$$

The absolute error in a sum or difference is the sum of the absolute errors in each measurement.

Error in a product or quotient

There are many circumstances in which we need to multiply or divide measurements by other measurements.

For instance, surface area is usually a multiple of two measurements.

$$\text{Surface area}_{\text{rectangle}} = \text{length} \times \text{width}$$

When multiplying or dividing two measurements that have an uncertainty associated with them, we always **add** the fractional or percentages errors in both cases. Percentage errors have been used below.

$$A \times B = (AB) \pm (AB \times \%\ error\ AB) \quad \text{and} \quad A \div B = \left(\frac{A}{B}\right) \pm \left(\frac{A}{B} \times \%\ error\ \frac{A}{B}\right)$$

Example 1

Calculate the area of a rectangle and its associated absolute error from the following data:

Length = 14.26 ± 0.02cm, Width = 5.94 ± 0.02cm

Solution

$$\%\text{ error in area} = \%\text{ error length} + \%\text{ error width}$$

$$\%\text{ error length} = \frac{\text{absolute error length}}{\text{length}} \times 100 = \frac{0.02}{14.26} = 0.14\%$$

$$\%\text{ error width} = \frac{\text{absolute error width}}{\text{width}} \times 100 = \frac{0.02}{5.94} = 0.34\%$$

$$\%\text{ error for Area} = 0.14\% + 0.34\% = 0.48\% \text{ (always add the errors)}$$

$$\begin{aligned}\text{Absolute Error for Area} &= (\text{Area}) \pm (\text{Area} \times \%\text{ error of Area}) \\ &= (14.26 \times 5.94) \pm (14.26 \times 5.94) \times 0.48\% \\ &= 84.7 \pm 0.4\text{cm}^2\end{aligned}$$

Error in a power

If one quantity is raised to a power then the percentage error is added as many times as the power. eg, a value raised to a power of 3 would have its percentage error trebled. See next page for an example of a percentage error using powers.

Example 2

For a sphere of radius 5.0 ± 0.1 cm, calculate the volume of the sphere and its associated error.

Solution

$\text{Volume} = \frac{4}{3} \times \pi \times (\text{radius})^3$ Constants $\frac{4}{3} \times \pi$ are ignored for errors

$\%$ error in volume $= 3 \times \%$ error in radius The radius is cubed, $\therefore$ error is $\times 3$

If radius was squared, error would be $\times 2$

If radius was quadrupled, error would be $\times 4$

$$= 3 \times \frac{\text{absolute error radius}}{\text{radius}} \times 100$$

$$= 3 \times \frac{0.1}{5} \times 100 = 6\%$$

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 5^3 = 524\text{cm}^3$$

$$\text{Absolute error in volume} = \text{volume} \times \% \text{ error volume}$$
$$= 524 \times 6\%$$
$$= 31 \text{ cm}^3$$

$\therefore$ Volume $= 524 \pm 31 \text{ cm}^3$

Exercise

1. If $A = 6.0 \pm 0.1$cm, and $B = 3.4 \pm 0.2$cm, calculate:

(a) A + B, (b) A – B (c) A × B (d) A ÷ B

2. The mass of a square cube was found to be 60.7 ± 0.05 g. A side length of this cube = 1.5 ± 0.05 cm.

Calculate the density of the material and its associated error. (Density = Mass/Volume)

3. A cylinder has a height of 10.3 ± 0.05 cm. Its radius is 4.5 ± 0.05 cm.

Calculate the volume of the cylinder and its associated error? Volume $V = \pi r^2 h$

Answers

1. (a) 9.4 ± 0.3cm (b) 2.6 ± 0.3cm (c) 20.4 ± 1.54cm (d) 1.76 ± 0.13cm 2. $18 \pm 2.6 \text{ g.cm}^{-3}$

3. $655 \pm 18 \text{ cm}^3$