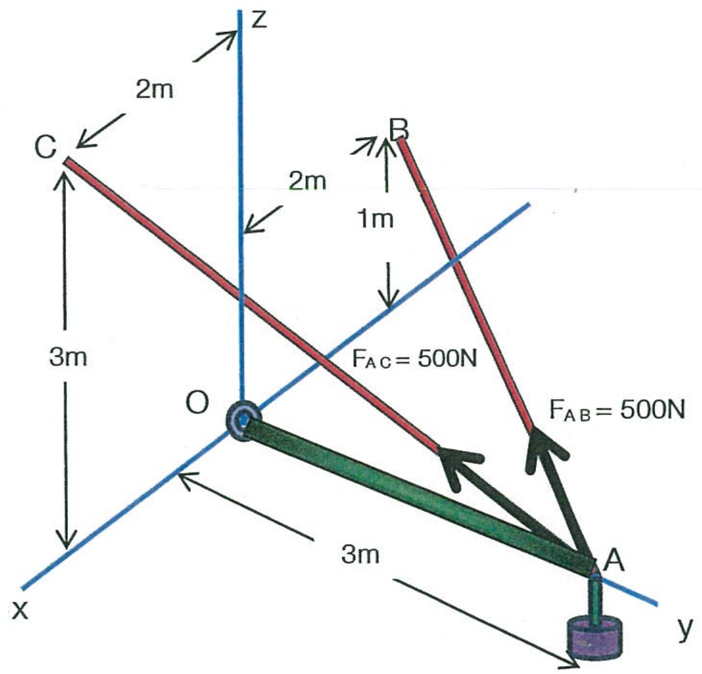
RMIT UNIVERSITY

WORKED SOLUTIONS

ENST2.3:

FORCES IN 3 DIMENSIONS

Question Determine (a) the vector forces in the cables F_{AB} and F_{AC}, (b) the resultant vector force F of F_{AB} and F_{AC}. (c) the scalar projection of F along the beam AO, (d) the perpendicular vector component of F to the beam AO, and (e) the angle between the cables AB and AC.



Solution

(a) Position vectors first

$\vec{OA} = (0,3,0)$ $\vec{OB} = (-2,0,1)$
$\vec{OC} = (2,0,3)$

$\vec{AB} = (-2,-3,1)$ and

$\hat{AB} = \frac{1}{\sqrt{14}}(-2,-3,-1)$

$\vec{AC} = (2,-3,3)$ and

$\hat{AC} = \frac{1}{\sqrt{22}}(2,-3,3)$

$$\vec{F}_{AB} = |\vec{F}_{AB}|\ \hat{AB} = 500\frac{1}{\sqrt{14}}(-2,-3,-1) \approx (-267,-401,134)\text{N}$$

$$\vec{F}_{AC} = |\vec{F}_{AC}|\ \hat{AC} = 500\frac{1}{\sqrt{22}}(2,-3,3) \approx (213,-320,320)\text{N}$$

(b) $$\vec{F} = \vec{F}_{AB} + \vec{F}_{AC} = (-267,-401,134)+(213,-320,320) = (-54,-721,454)\text{ N}$$

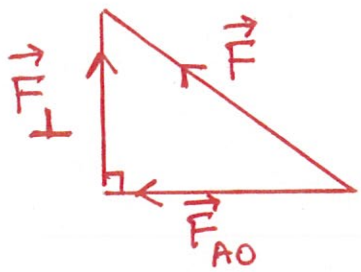
(c) Scalar component of $\vec{F}$ along beam $\vec{AO}$: $\vec{F}_{AO} = \vec{F} \bullet \hat{AO}$

where $\hat{AO} = \dfrac{\vec{AO}}{|\vec{AO}|} = \dfrac{(0,-3,0)}{\sqrt{9}} = (0,-1,0)$

Hence $\vec{F}_{AO} = (-54,-721,454)\bullet(0,-1,0) = 721\text{ N}$

Note: Since the result is positive $\vec{F}$ has the same sense of direction as $\vec{AO}$

(d) Perpendicular component of $\vec{F}$ to beam $\vec{AO}$



Now $\vec{F}_{AO} = |\vec{F}_{AO}|\,\hat{AO} = 721\,(0,-1,0)$

$\vec{F}_{AO} = (0,-721,0)\text{ N}$

$\vec{F} = \vec{F}_{\perp} + \vec{F}_{AO} \Rightarrow \vec{F}_{\perp} = \vec{F} - \vec{F}_{AO}$

$\therefore \vec{F}_{\perp} = (-54,-721,454) - (0,-721,0) = (-54,0,454)\text{ N}$

(e) Angle between 2 vectors is given by dot product

$\vec{AB} \bullet \vec{AC} = |\vec{AB}||\vec{AC}|\cos\theta \Rightarrow \theta = \cos^{-1}\left(\dfrac{\vec{AB}\bullet\vec{AC}}{|\vec{AB}|\,|\vec{AC}|}\right)$

$$\therefore \theta = \cos^{-1}\frac{(-2,-3,1)\bullet(2,-3,3)}{\sqrt{14}\times\sqrt{22}} = 63^\circ$$