RMIT UNIVERSITY

ENDY1.4
MOTION: CONSTANT ACCELERATION

Equations with uniform acceleration: horizontal motion

We have already used the formula $v = \frac{s}{t}$ to describe motion where the velocity is constant. This formula cannot be used in situations where there is uniform accelerated motion.

Recall from graphs of motion that the uniform acceleration 'a' of an object is given by the gradient of a velocity-time graph:

$$acceleration \;\; a = \frac{Rise}{Run} = \frac{change\,in\,velocity}{time\;taken} = \frac{\Delta v}{t} = \frac{v_f - v_i}{t} \qquad \therefore \; at = v_f - v_i$$

Since $a \times t = v_f - v_i$

$\boxed{v_f = v_i + at}$ Similar mathematical gymnastics can be used to show that:

$x = v_i t + \frac{1}{2} a t^2$	$x = v_f t - \frac{1}{2} a t^2$	$v_f^{\,2} = v_i^{\,2} + 2ax$	$x = \frac{1}{2}(v_i + v_f)\, t$

where x = displacement(m), v_i = initial velocity(ms^{-1}), v_f = final velocity (ms^{-1}),
t = time taken (s)

Other symbols may be used to denote x *(s)*, 'v_i' (u , v_o) and v_f (v).

Note: It is often necessary to specify right or left as the positive or negative direction when doing these problems since we are dealing with vector quantities.

Example
A truck has an acceleration of $0.8ms^{-2}$. Find (a) the time taken and (b) the distance travelled for the truck to reach a velocity of $20ms^{-1}$ from a starting velocity of $3ms^{-1}$.

(a) List the data first: $v_i = 3$, $v_f = 20$, $a = 0.8$, $t = ?$

Now find the appropriate equation that includes this data: $v_f = v_i + at$

From which $t = \frac{v_f - v_i}{a} = \frac{20-3}{0.8} = 21.3s$

(b) List the data first: $v_i = 3$, $v_f = 20$, $a = 0.8$, $x = ?$

Now find the appropriate equation that includes this data: $v_f^{\,2} = v_i^2 + 2ax$

From which $x = \frac{v_f^{\,2} - v_i^{\,2}}{2a} = \frac{20^2 - 3^2}{2 \times 0.8} = 244.4m$

Vertical motion under gravity

The acceleration of a falling object near the Earth's surface is 9.8ms^{-2}. For instance, a coin that is dropped from rest will have a velocity of 9.8ms^{-1} after 1 second, 19.6ms^{-1} after 2 seconds, and so on. The quantity, 'g', can also be used when describing gravitational acceleration. For example, an astronaut will experience an acceleration of 39.2ms^{-2}, or 4g, at take-off.

Since the acceleration of a freely falling object is uniform (constant), we can use the same equations as we did with horizontal motion.

Note: It is often necessary to specify up or down as the positive or negative direction when doing these problems as we are dealing with vector quantities.

Example 1

A construction worker accidentally knocks a brick from a building so that it falls vertically a distance of 50m to the ground. Calculate (a) the time taken for the brick to reach the ground, (b) the speed of the brick as it hits the ground.

(a) List the data first: $v_i = 0$ (at rest), $x = 50$, $a = 9.8$, $t = ?$

Now find the appropriate equation that includes this data: $x = v_i t + \frac{1}{2} a t^2$

$$x = v_i t + 0.5 \times 9.8 \times t^2 \Rightarrow 50 = 0 + 0.5 \times 9.8 \times t^2 \Rightarrow t = 3.2s$$

(b) List the data first: $v_i = 0$ (at rest), $x = 50$, $a = 9.8$, $t = 3.2$, $v_f = ?$

Now find the appropriate equation that includes this data: $v_f = v_i + at$

$$v_f = v_i + at = 0 + 9.8 \times 3.2 \Rightarrow v_f = 31ms^{-1}$$

Example 2

A ball is thrown up into the air with a velocity of 30ms^{-1}. Find (a) the maximum height reached by the ball, (b) the time taken for the ball to reach its maximum height.

Take **up** as positive, and **down** as negative.

(a) List the data first: $v_i = +30$ (velocity is **up**), $v_f = 0$ (ball is stationary at top of flight), $a = -9.8$ (acceleration is always **down**), $x = ?$

Now find the appropriate equation that includes this data: $v_f^{\,2} = v_i^2 + 2ax$

$$0 = 30^2 + 2 \times -9.8 \times x \Rightarrow x = 46m$$

(b) List the data first: $v_i = +30$ (velocity is **up**), $v_f = 0$ (ball is stationary at top of flight), $a = -9.8$ (acceleration is always **down**), $x = 45$, $t = ?$

Now find the appropriate equation that includes this data: $v_f = v_i + at$

$0 = 30 + -9.8 \times t \Rightarrow t = 3.1s$

Note: by symmetry, the ball will also take 3.1 seconds to return to the ground, assuming that air resistance is ignored. Total time of flight = 6.2 seconds.

Exercise

Use g = $9.8ms^{-2}$

1. A car, starting from rest, moves with an acceleration of $2ms^{-2}$. Find (a) the velocity at the end of 20 seconds, and (b) the distance covered in that time.

2. With what uniform acceleration does a spacecraft, starting from rest, cover 1000 metres in 10 seconds?

3. A cyclist, starting from rest, moves with an acceleration of $3ms^{-2}$. (a) In what time will it reach a velocity of 30m/s, and (b) what distance does the cyclist cover in this time?

4. An object starts with a velocity of 100m/s and decelerates (slows down) at $2ms^{-2}$. (a) When will its velocity be zero, and (b) how far will it have gone?

5. A book is knocked off a bench and falls vertically to the floor. If the book takes 1.0s to fall to the floor, calculate:
   a. its speed as it lands.
   b. the height from which it fell.
   c. the distance it falls during the first 0.5s.
   d. the distance it falls during the final 0.5s.

6. A champagne cork travels vertically into the air. It takes 4.0s to return to its starting position.
   a. How long does the cork take to reach its maximum height?
   b. What was the maximum height reached by the cork?
   c. How fast was the cork travelling initially?
   d. What the speed of the cork as it returned to its starting point?
   e. Describe the acceleration of the cork at each of these times after its launch: (i) 1.0s (ii) 2.0s (iii) 3.0s.

Answers

1. (a) 40 ms^{-1} (b) 400m 2. 20 ms^{-2} 3. (a) 10s (b) 150m 4. (a) 50s (b) 2500m

5. (a) $9.8ms^{-1}$ (b) 4.9m (c) 1.2m (d) 3.7m 6. (a) 2.0s (b) 19.6m (c) $19.6ms^{-1}$ (d) $19.6ms^{-1}$

(e) (i) $9.8ms^{-2}$ down (ii) $9.8ms^{-2}$ down (iii) $9.8ms^{-2}$ down